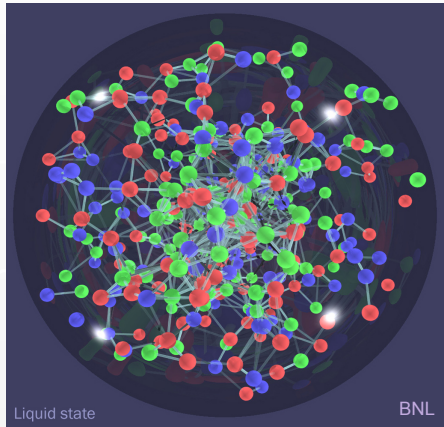


Gauge/Gravity



Duality



Joseph Polchinski



Fermilab, 2/24/10

Which of these interactions doesn't belong?

a) Electromagnetism

b) Weak nuclear

c) Strong nuclear

d) Gravity

The gauge interactions:

Maxwell's equations
+ Dirac equation
+ Quantum field theory

Quantum electrodynamics

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

$$i\gamma^\mu D_\mu \psi = m\psi$$



The gauge interactions:

Maxwell's equations

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

+ Dirac equation

$$i\gamma^\mu D_\mu \psi = m\psi$$

+ Quantum field theory

Quantum electrodynamics

+ Some indices

e.g. $F^{\mu\nu} \rightarrow F_{ij}^{\mu\nu}$ $2 \times 2, 3 \times 3$

+ Higgs scalar

Standard Model

The gauge interactions:

Maxwell's equations

$$\partial_\mu F^{\mu\nu} = j^\nu, \quad \partial_\mu \tilde{F}^{\mu\nu} = 0$$

+ Dirac equation

$$i\gamma^\mu D_\mu \psi = m\psi$$

+ Quantum field theory

Quantum electrodynamics

+ Some indices

e.g. $F^{\mu\nu} \rightarrow F_{ij}^{\mu\nu}$ $2 \times 2, 3 \times 3$

+ Higgs scalar

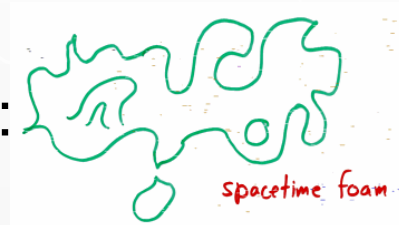
Standard Model

Well understood: any calculation can be reduced to an algorithm (nontrivial!).

Gravity

- Spacetime is dynamical, and quantum mechanical.

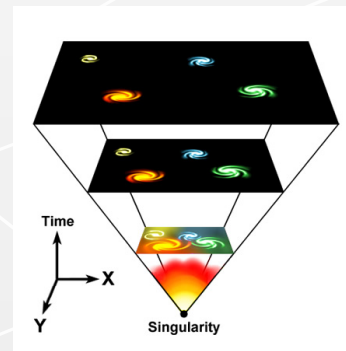
- UV divergences/spacetime foam:



- Quantum black hole puzzles-
entropy and information

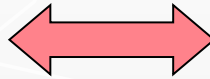


- Spacetime singularities
- Initial conditions



The remarkable discovery is that these two kinds of theory are dual.

gauge theory



Gravity

- The same theory, expressed in different variables
- Different classical limits of a single quantum theory (e.g. waves vs. particles).

Outline:

- Derivation 1: A crazy idea
- Derivation 2: Black branes and D-branes
- What this teaches us about gauge theory
- What it teaches us about gravity



I. Question: Could the spin-2 graviton be a bound state of two spin-1 gauge bosons?



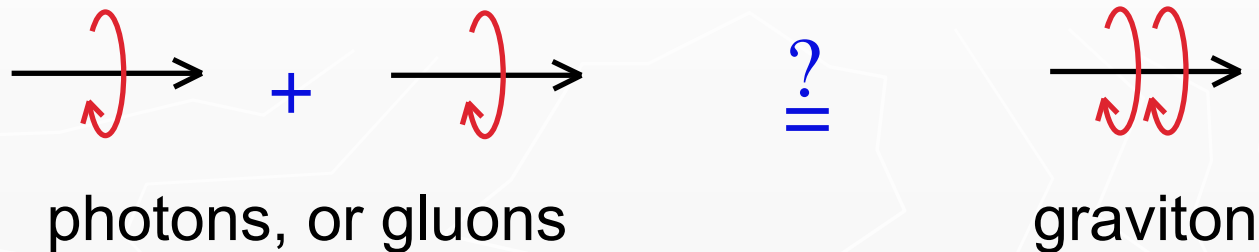
photons, or gluons

$\stackrel{?}{=}$



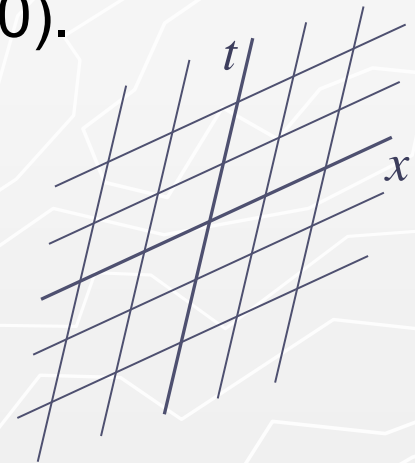
graviton

I. Question: Could the spin-2 graviton be a bound state of two spin-1 gauge bosons?



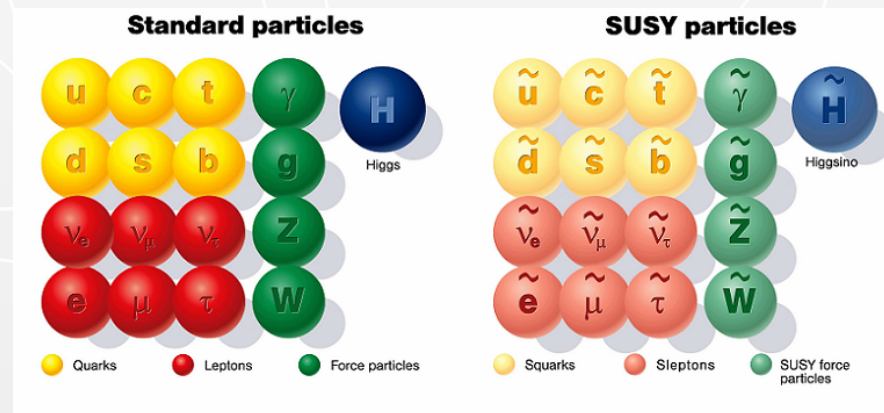
No-go theorem! (Weinberg & Witten, 1980).

Heuristic: gauge theories are invariant under *special* relativity. Massless spin 2 requires *general* coordinate invariance. There is a mismatch of observables.



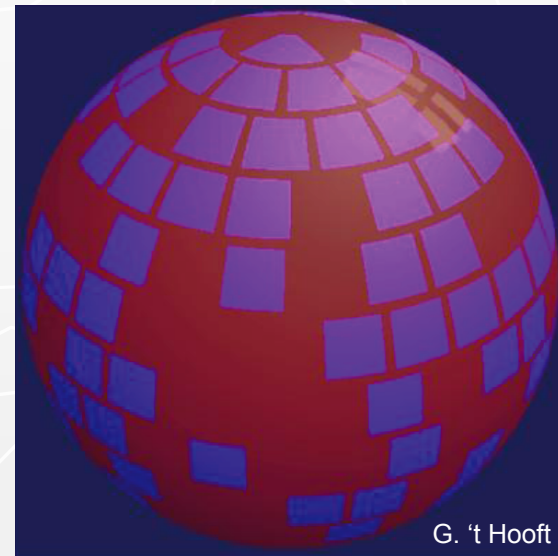
To prove such a no-go theorem one must make assumptions, and sometimes the most obvious assumptions turn out to be wrong, and lead to powerful new ideas.

Example: the Coleman-Mandula theorem, which ruled out the unification of spacetime and internal symmetries. Assumption: conserved charges are bosonic. Exception: supersymmetry!

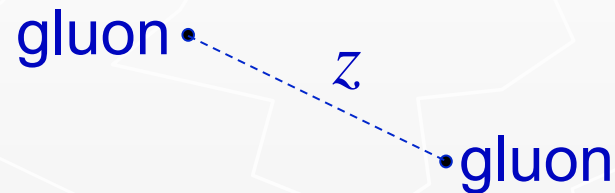


Hidden assumption: the graviton moves in the same space as the gauge bosons it is made of!

The *holographic principle* ('t Hooft, Susskind): quantum gravity in any space can be formulated in terms of degrees of freedom living on the *boundary* of the space. This is motivated by the Bekenstein-Hawking black hole entropy, $S_{\text{BH}} = A/4l_{\text{Planck}}^2$.



The holographic principle suggests that gauge theory in $3+1$ dimensions should somehow give rise to gravity in $4+1$ dimensions. To get gravity in $3+1$ we should start with gauge theory in $2+1$. But where does the extra dimension come from?

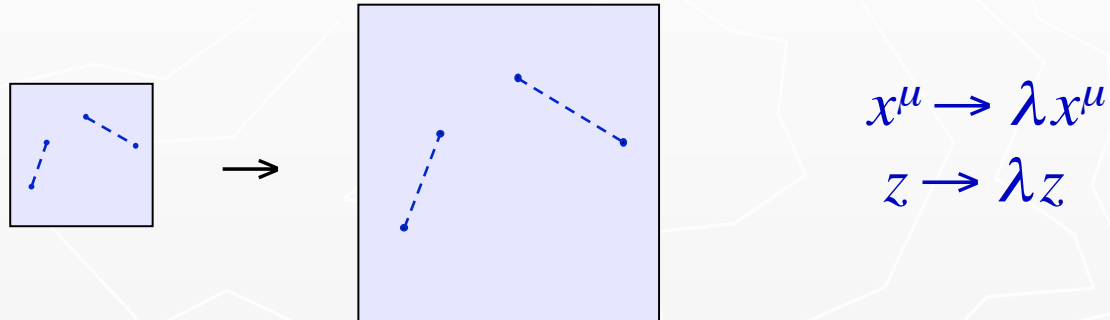


In QCD, sometime the interactions between gluon pairs are approximately local in the *separation* z (color transparency, BFKL). This behaves crudely like a fifth coordinate, where the usual four come from the center-of-mass.

‘Emergent dimension’

The shape of the emergent spacetime:

QCD at high energy has approximate symmetry under scaling all lengths:



This determines the metric, up to overall radius L :

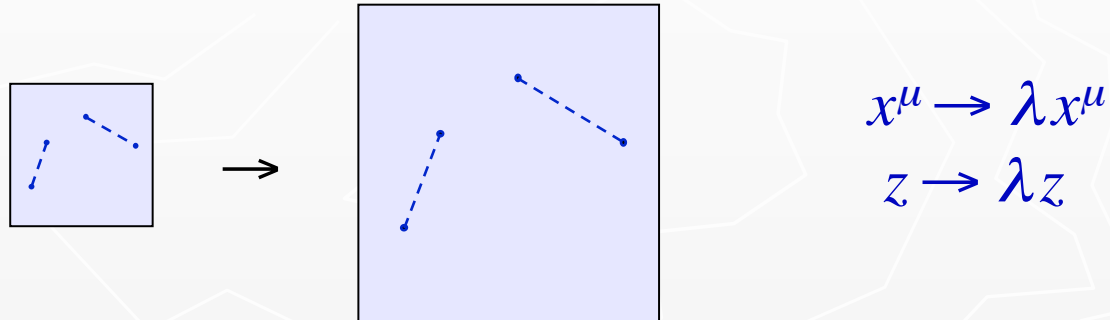
$$ds^2 = L^2(-dt^2 + dw^2 + dx^2 + dy^2 + dz^2)/z^2$$

Anti-de Sitter space (AdS). de Sitter space ($z \leftrightarrow t$) expands exponentially, while AdS is 'warped.'

'AdS/CFT'

The shape of the emergent spacetime:

QCD at high energy has approximate symmetry under scaling all lengths:



This determines the metric, up to overall radius L :

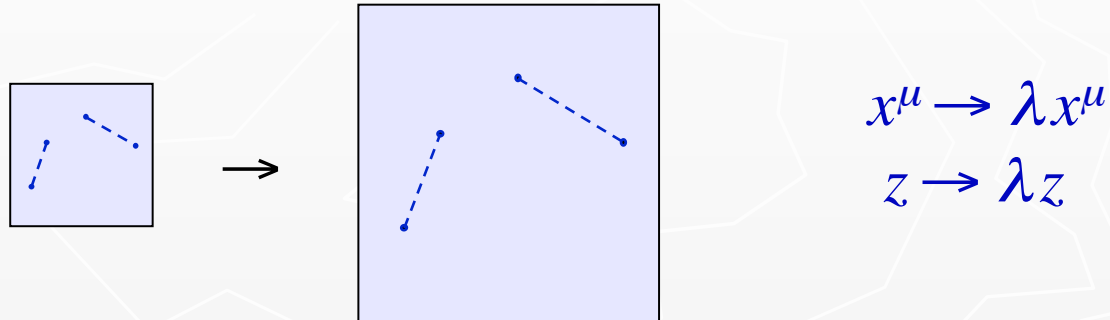
$$ds^2 = L^2(-dt^2 + dw^2 + dx^2 + dy^2 + dz^2)/z^2$$

Anti-de Sitter space (AdS). de Sitter space ($z \leftrightarrow t$) expands exponentially, while AdS is 'warped.'

'AdS/CFT'

The shape of the emergent spacetime:

QCD at high energy has approximate symmetry under scaling all lengths:



This determines the metric, up to overall radius L :

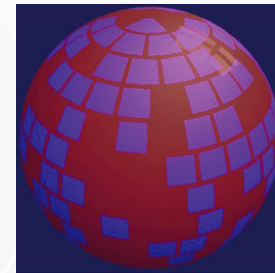
$$ds^2 = L^2(-dt^2 + dw^2 + dx^2 + dy^2 + dz^2)/z^2$$

Anti-de Sitter space (AdS). de Sitter space ($z \leftrightarrow t$) expands exponentially, while AdS is 'warped.'

*This is 4+1 gravity from 3+1 gauge theory. To get 3+1 gravity, start with 2+1 gauge theory.

Two more ingredients:

- **Large N** (size of gauge matrices), to have enough states. 't Hooft (1974) showed that one gets an interesting limit if one replaced the 3 colors of QCD with a large number N .

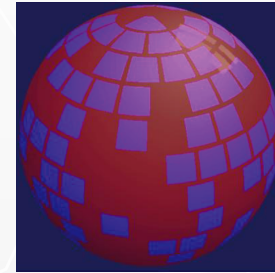


- **Strong coupling**, to get interesting bound states.

Under these conditions, gravity emerges! (Heemskerk, Penedones, JP & Sully, arXiv/0907.0151).

Two more ingredients:

- **Large N** (size of gauge matrices), to have enough states. 't Hooft (1974) showed that one gets an interesting limit if one replaced the 3 colors of QCD with a large number N .

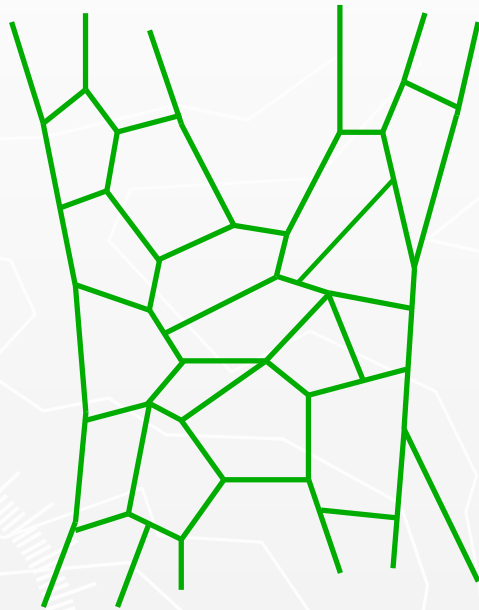


- **Strong coupling**, to get interesting bound states.

Under these conditions, gravity emerges! (Heemskerk, Penedones, JP & Sully, arXiv/0907.0151).

But what about string theory?

Along with gravity come strings, branes, M theory...



't Hooft (1974): large N = planar graphs. Strong coupling limit becomes a string.

II. Black branes and D-branes

Original derivation of gauge/gravity duality driven by problems of black hole quantum mechanics - the *entropy puzzle* and the *information paradox*.



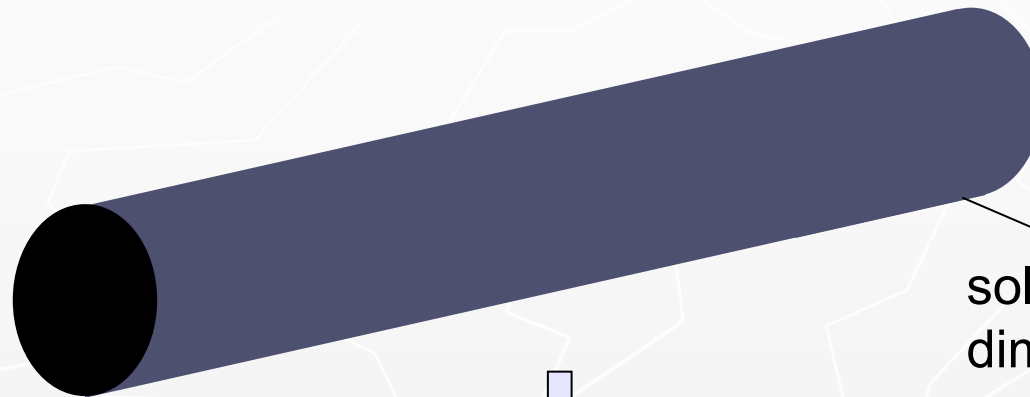
- Black hole entropy $S_{\text{BH}} = A/4l_{\text{Planck}}^2$ - what does this mean?
- Originally, an analogy: in classical GR, the total horizon area, like the entropy, is nondecreasing.
- With the discovery of Hawking radiation (1974), this is more than an analogy: only the *sum*

$$S_{\text{total}} = S_{\text{BH}} + S_{\text{ordinary}}$$

is nondecreasing.

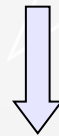
- But thermodynamics is a phenomenology, stat mech is the full story. What states is S_{BH} counting?

Strominger and Vafa (1996): imagine adiabatically reducing the gravitational coupling, so that a black hole is no longer black. For some string theory black branes,



solution to higher dimensional GR

one finds that they turn into D-branes,

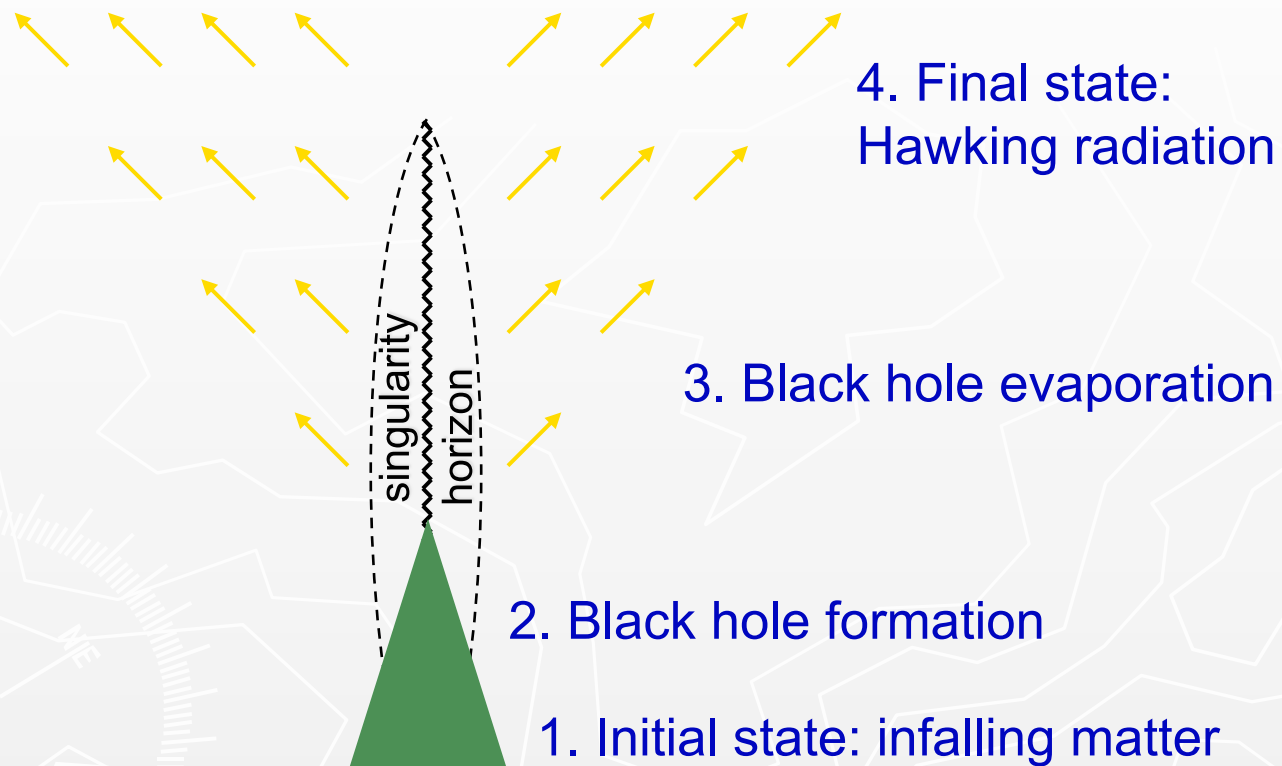


the strings of string theory

D-branes, stringy defects

whose states we can count, and we do find S_{BH} .

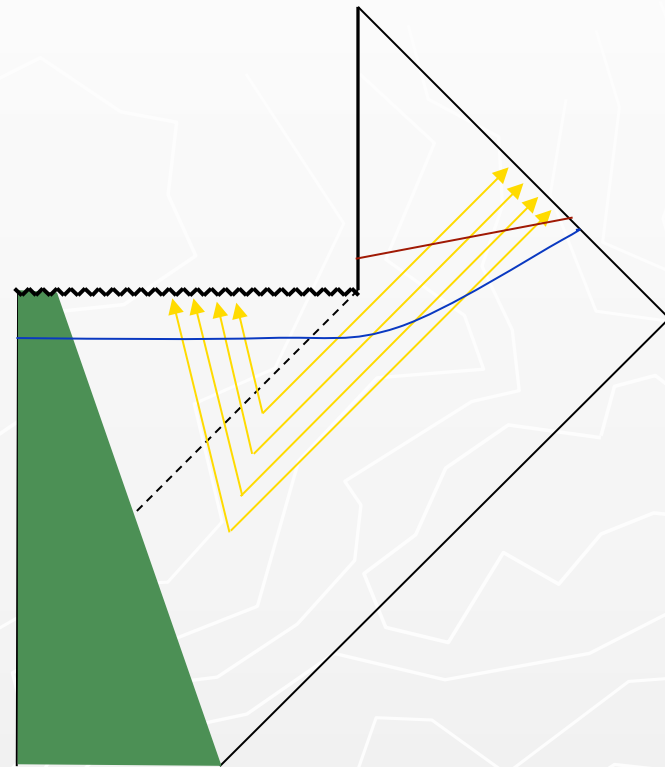
The information paradox: Hawking's thought experiment (1976).



Repeat many times, with same initial state and all possible measurements on the final state.

Conclusion: initial pure state evolves to final mixed state (density matrix); ordinary QM is pure \rightarrow pure.

Each Hawking particle
is correlated with one
behind the horizon;
when evaporation is
complete these
correlations are lost.



No trivial resolution; the alternative seems to be a radical breakdown of spacetime locality.

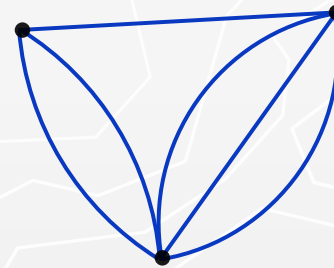
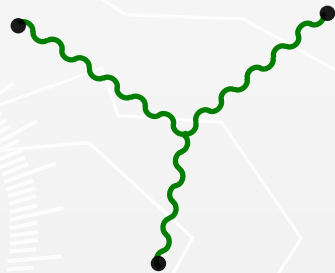
Repeat Hawking's calculation with



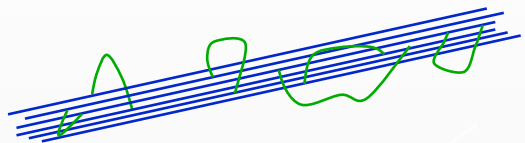
vs.



Result: very different calculations give identical answers in many cases.



Explained by Maldacena; Gubser, Klebanov, Polyakov; Witten (1997-8) as gauge/gravity duality.



D-branes and strings

low
energy
limit

$\mathcal{N}=4$ $SU(N)$ gauge
theory

coupling
weak strong



black hole or brane

low
energy
limit

coupling
weak strong

IIB superstring with
 $AdS_5 \times S^5$ b.c.

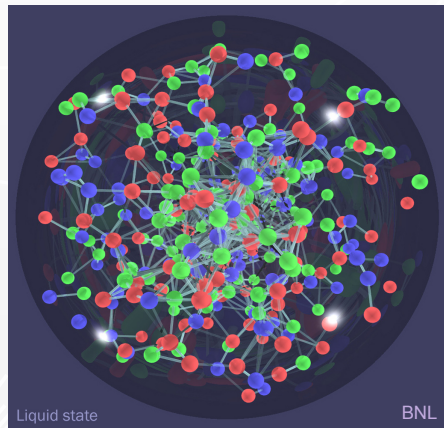
$\mathcal{N}=4$ supersymmetric
 $SU(N)$ gauge theory = IIB superstring with
 $AdS_5 \times S^5$ b.c.

- 1-to-1 mapping of spectra, transition amplitudes
- *many* checks, still no complete derivation
- many generalizations,

less symmetric gauge
theory = string theory in less
symmetric space

E.g., what happens if we heat the system up?

Gauge theory



Plasma

Gravity



Black hole
(highest entropy)

Different limits of the same object

So, what do we learn from

gauge theory = gravity/string theory ?

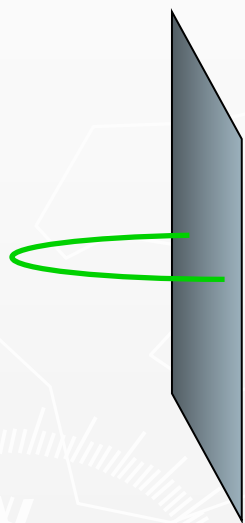
Information flows in both directions:

gauge theory ← gravity/string theory

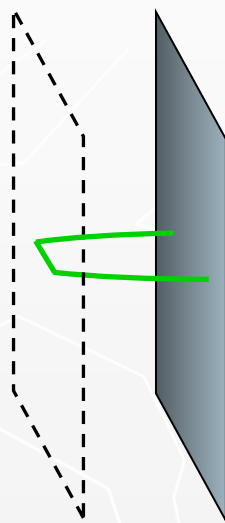
gauge theory → gravity/string theory

gauge theory  gravity/string theory

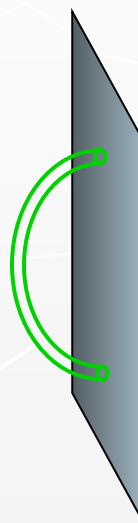
Enables us to calculate things at strong coupling -



Coulomb force $\sim g$ rather than g^2



Confinement
when scale sym-
metry broken



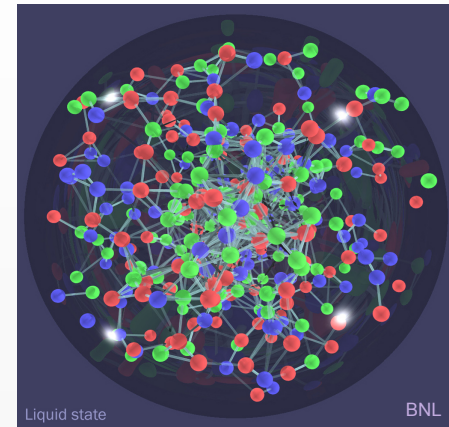
Anomalous
dimensions
 $\sim (g^2 N_c)^{1/4}$



Free energy
= $3/4$ of free-
field value.

This is for the $\mathcal{N} = 4$ theory, how about QCD?

The Relativistic Heavy Ion Collider has been making the quark-gluon plasma state. Surprise: it is much more liquid-like than gas-like --- that is, the interactions are *strong*.



QCD does not precisely fit the conditions for having a gravity dual, but it's not too far off, and much better than the ideal gas approximation. So calculate using the black hole description!



Successes:

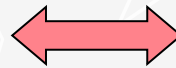
- Viscosity/entropy $\sim 1/4\pi$ vs, 0.02–0.12 measured.
(Kovtun, Son, Starinets, 1994)
- Free energy $\sim 0.8 \times$ free field value.
- Jet quenching properties
- Heavy quark and heavy meson drag and screening.
- Higher order relativistic hydrodynamics
- ...

Black hole is the 'spherical cow' for heavy ion collisions (K. Rajagopal).

Less useful for particle physics applications of QCD because in its current state it can't describe the asymptotically free/partonic regime.

Main particle physics consequence: the duality

Strong gauge
dynamics



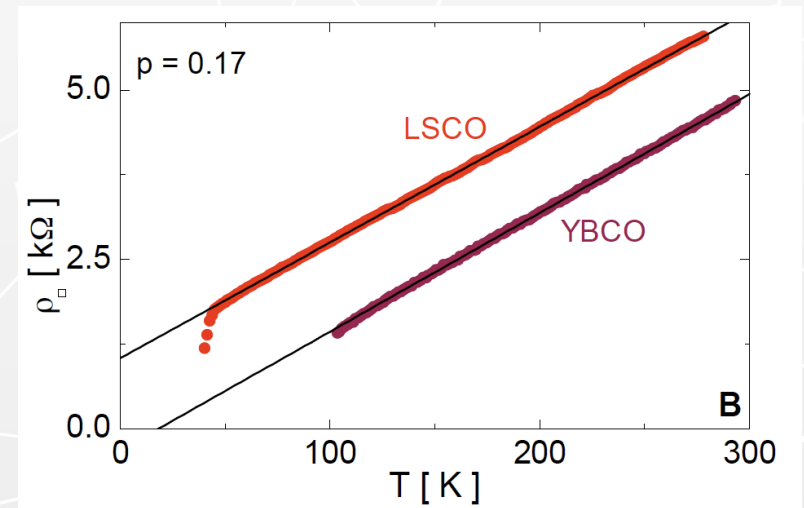
Randall-Sundrum
models

New direction: strongly coupled phases in condensed matter systems.

Naturalness: the 'Planck scale' of CM is $\sim 1 \text{ eV} \sim 10^4 \text{ K}$.

Conductors have much lower energy charged degrees, and they are common, so there must be a 'natural' low energy theory. This is the Landau-Fermi liquid model, the 'Standard Model' of conductors.

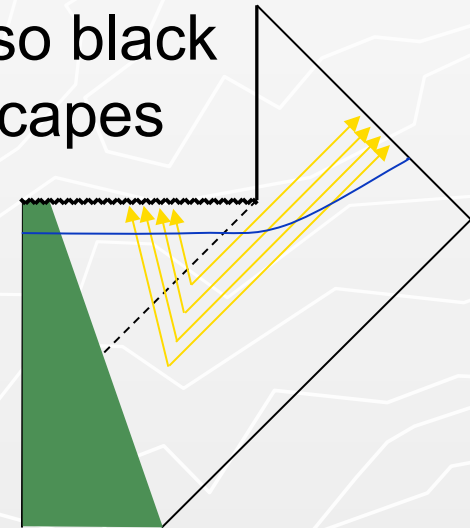
'BSM' physics has been found in high- T_c cuprates, heavy fermion systems, etc., not understood. Seems to involve strong scale-invariant dynamics.



gauge theory  gravity/string theory

Resolves black hole puzzles and paradoxes:

- Black holes behave like thermal systems because that is exactly what they are in the dual variables.
- Gauge theory plasmas satisfy ordinary quantum mechanics, pure states to pure states, so black holes must as well -- the information escapes with the Hawking radiation. Locality must break down radically; not so surprising in a holographic theory, details still not understood.



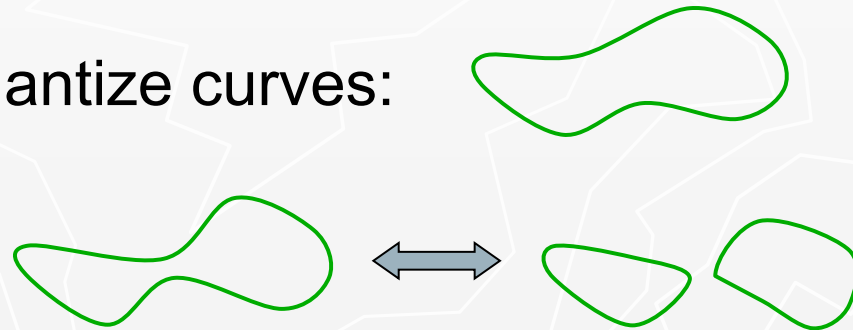
gauge theory  gravity/string theory

Gives a construction of the RHS: What is string theory?

QFT - quantize points: .

String theory - quantize curves:

Splitting
interaction:

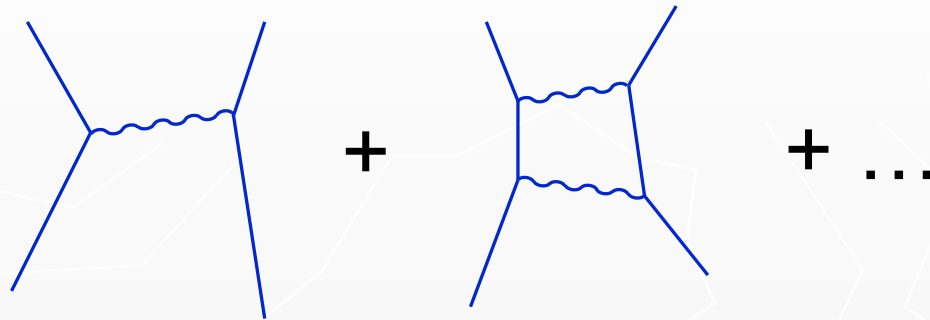


Gives gravity, solves UV problem, unifies gravity with gauge theory.

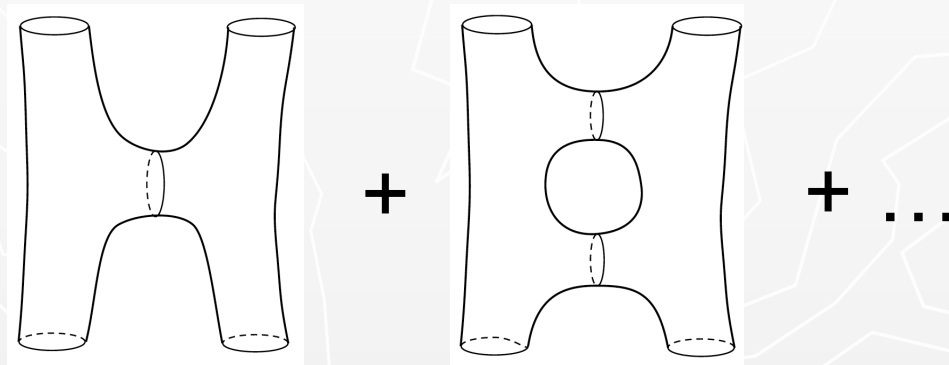
Limitation: perturbative only

Perturbation theory:

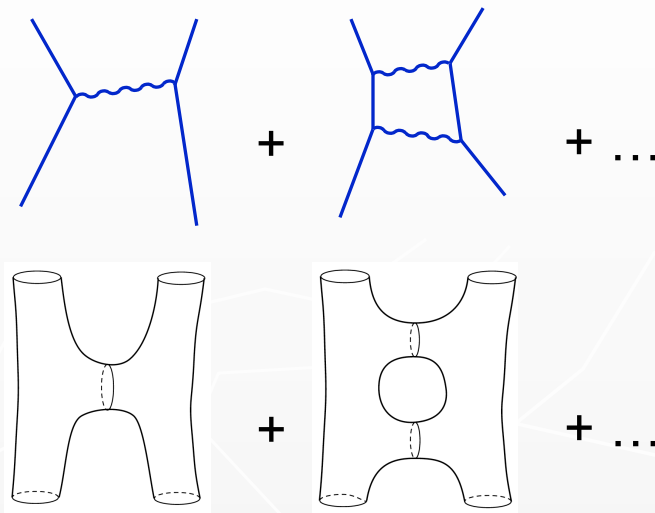
QFT:



String:




Contains a lot of information, but misses a lot.
Doesn't converge!



What are these approximations to?

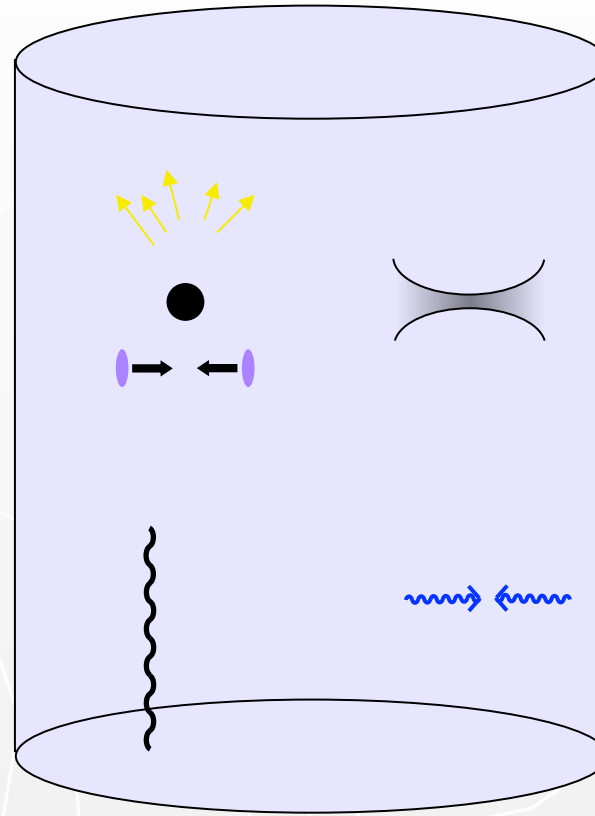
For QFT, this was answered by Ken Wilson: precise description is path integral + renormalization group.

For string theory, “gauge theory  string theory” now gives a *partial* answer, and points to the *holographic principle*.

The gauge theory on the boundary encodes a lot of the physics of quantum gravity:

Black hole formation and evaporation

Resolution of some spacetime singularities



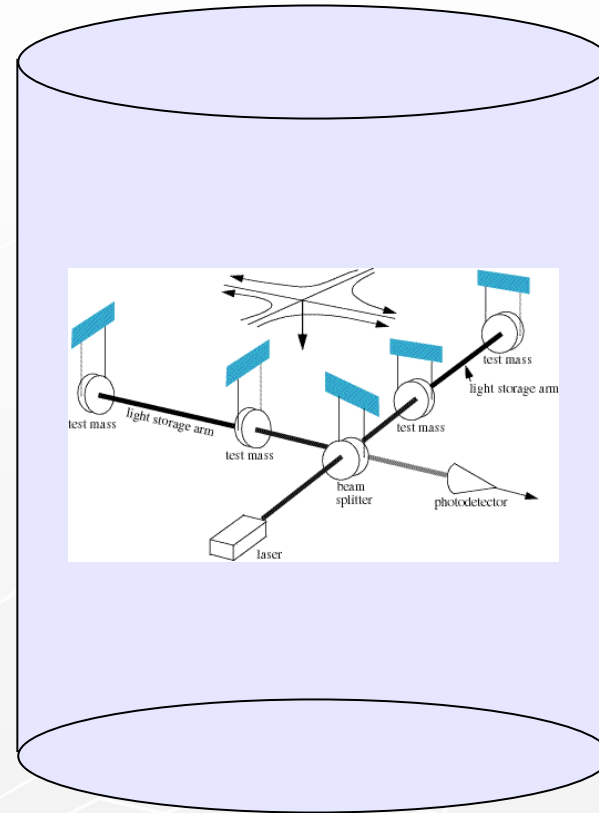
AdS

Spacetime topology change

Hyper-Planckian scattering

Almost background-independent, but the boundary is pinned down. In a holographic theory, the boundary is crucial.

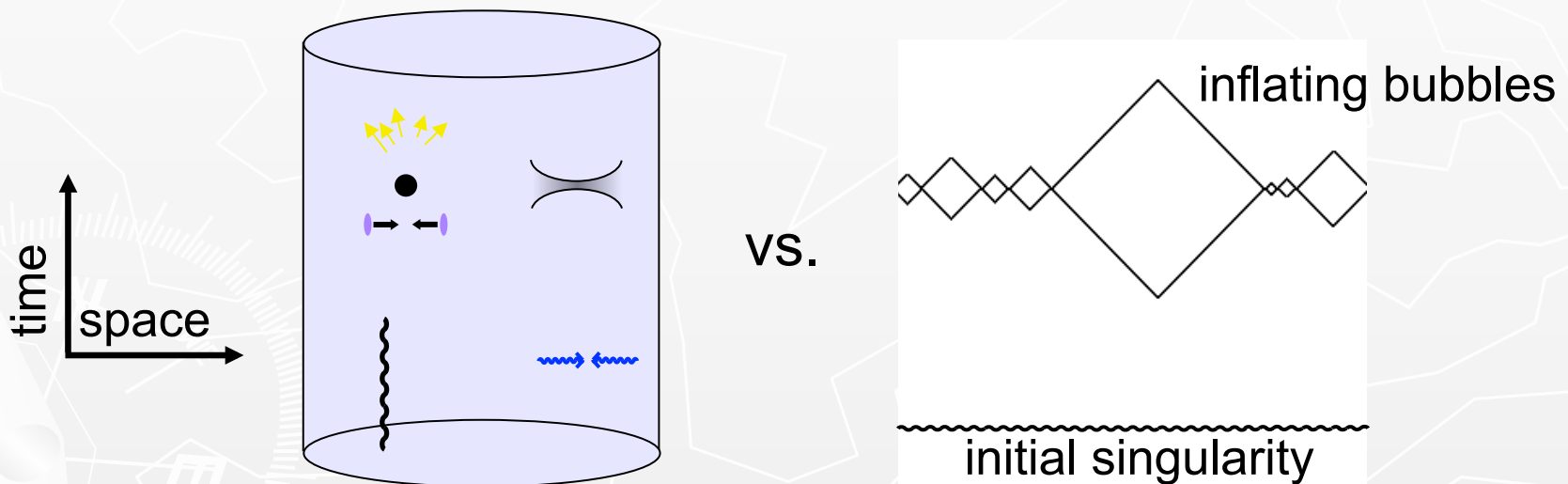
Hogan has suggested that holography will lead to increased gravitational fluctuations, perhaps visible at interferometers.



This is exciting, though there is not yet agreement that the effect is so large.

Cosmology is the frontier for these ideas

In a cosmological spacetime (de Sitter, FRW, chaotic inflation) the natural boundaries are past and future, and so it is *time* that would have to emerge... what does this mean?



Cosmology provides new puzzles and paradoxes, and a new window on the highest energies.

Conclusion

- Gauge/gravity duality means that string theory is not such a new thing, it was hidden all along in the structure of ordinary quantum field theories. It is a remarkable connection between different parts of physics.
- The idea that physics is holographic, that the fundamental variables are nonlocal and locality is emergent, is revolutionary, and we have much to understand.